

Preview

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What do you think? -

- How are measurements such as mass and volume different from measurements such as velocity and acceleration? ▾
- How can you add two velocities that are in different directions?

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Introduction to Vectors -

- Scalar - a quantity that has magnitude but no direction ▾
  - Examples: volume, mass, temperature, speed ▾
- Vector - a quantity that has both magnitude and direction ▾
  - Examples: acceleration, velocity, displacement, force

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### Vector Properties

- Vectors are generally drawn as arrows.
  - Length represents the magnitude
  - Arrow shows the direction
- Resultant - the sum of two or more vectors

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### Finding the Resultant Graphically

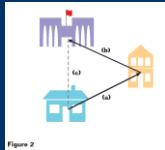


Figure 2

- Method
  - Draw each vector in the proper direction.
  - Establish a scale (i.e. 1 cm = 2 m) and draw the vector the appropriate length.
  - Draw the resultant from the tip of the first vector to the tail of the last vector.
  - Measure the resultant.
- The resultant for the addition of  $a + b$  is shown to the left as  $c$ .

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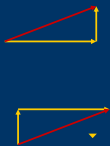
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### Vector Addition



- Vectors can be moved parallel to themselves without changing the resultant.
  - the red arrow represents the resultant of the two vectors

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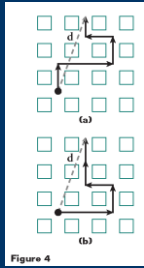
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### Vector Addition



- Vectors can be added in any order. ▾
  - The resultant (d) is the same in each case ▾
- Subtraction is simply the addition of the opposite vector.

Figure 4

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### Properties of Vectors

Click below to watch the Visual Concept.

Visual Concept

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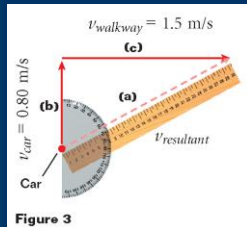
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### Sample Resultant Calculation



- A toy car moves with a velocity of .80 m/s across a moving walkway that travels at 1.5 m/s. Find the resultant speed of the car.

Figure 3

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### Now what do you think? ▾

- How are measurements such as mass and volume different from measurements such as velocity and acceleration? ▾
- How can you add two velocities that are in different directions?

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### What do you think? ▾

- What is one disadvantage of adding vectors by the graphical method? ▾
- Is there an easier way to add vectors?

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### Vector Operations ▾

- Use a traditional x-y coordinate system as shown below on the right. ▾
- The Pythagorean theorem and tangent function can be used to add vectors. ▾
  - More accurate and less time-consuming than the graphical method

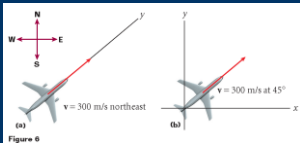


Figure 6

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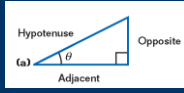
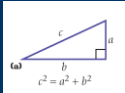
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Pythagorean Theorem and Tangent Function

**PYTHAGOREAN THEOREM FOR RIGHT TRIANGLES**  
 $c^2 = a^2 + b^2$   
 (length of hypotenuse)<sup>2</sup> = (length of one leg)<sup>2</sup> + (length of other leg)<sup>2</sup>

**DEFINITION OF THE TANGENT FUNCTION FOR RIGHT TRIANGLES**  
 $\tan \theta = \frac{\text{opp}}{\text{adj}}$       tangent of angle =  $\frac{\text{opposite leg}}{\text{adjacent leg}}$




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Vector Addition - Sample Problems

- 12 km east + 9 km east = ?  
 – Resultant: 21 km east
- 12 km east + 9 km west = ?  
 – Resultant: 3 km east
- 12 km east + 9 km south = ?  
 – Resultant: 15 km at 37° south of east
- 12 km east + 8 km north = ?  
 – Resultant: 14 km at 34° north of east

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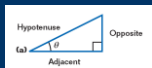
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Resolving Vectors Into Components

**DEFINITION OF THE SINE FUNCTION FOR RIGHT TRIANGLES**  
 $\sin \theta = \frac{\text{opp}}{\text{hyp}}$       sine of an angle =  $\frac{\text{opposite leg}}{\text{hypotenuse}}$

**DEFINITION OF THE COSINE FUNCTION FOR RIGHT TRIANGLES**  
 $\cos \theta = \frac{\text{adj}}{\text{hyp}}$       cosine of an angle =  $\frac{\text{adjacent leg}}{\text{hypotenuse}}$




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### Resolving Vectors into Components

- Opposite of vector addition
- Vectors are resolved into x and y components
- For the vector shown at right, find the vector components  $v_x$  (velocity in the x direction) and  $v_y$  (velocity in the y direction). Assume that that the angle is  $20.0^\circ$ .
- Answers:
  - $v_x = 89 \text{ km/h}$
  - $v_y = 32 \text{ km/h}$

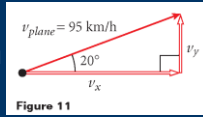


Figure 11

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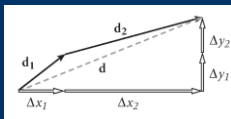
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### Adding Non-Perpendicular Vectors



- Four steps
  - Resolve each vector into x and y components
  - Add the x components ( $x_{total} = \Delta x_1 + \Delta x_2$ )
  - Add the y components ( $y_{total} = \Delta y_1 + \Delta y_2$ )
  - Combine the x and y totals as perpendicular vectors

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### Adding Vectors Algebraically

Click below to watch the Visual Concept.

Visual Concept

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**Classroom Practice**

- A camper walks 4.5 km at  $45^\circ$  north of east and then walks 4.5 km due south. Find the camper's total displacement.
- Answer
  - 3.4 km at  $22^\circ$  S of E

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**Now what do you think?**

- Compare the two methods of adding vectors.
- What is one advantage of adding vectors with trigonometry?
- Are there some situations in which the graphical method is advantageous?

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**What do you think?**

- Suppose two coins fall off of a table simultaneously. One coin falls straight downward. The other coin slides off the table horizontally and lands several meters from the base of the table.
  - Which coin will strike the floor first?
  - Explain your reasoning.
- Would your answer change if the second coin was moving so fast that it landed 50 m from the base of the table? Why or why not?

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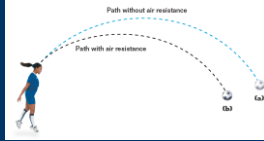
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Projectile Motion

- Projectiles: objects that are launched into the air
  - tennis balls, arrows, baseballs, wrestlers
- Gravity affects the motion
- Path is parabolic if air resistance is ignored
- Path is shortened under the effects of air resistance



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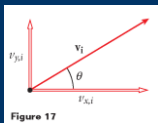
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Components of Projectile Motion



- As the runner launches herself ( $v_i$ ), she is moving in the x and y directions.



$$v_{x,i} = v_i \cos \theta$$

$$v_{y,i} = v_i \sin \theta$$

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Analysis of Projectile Motion

- Horizontal motion
  - No horizontal acceleration
  - Horizontal velocity ( $v_x$ ) is constant.
- How would the horizontal distance traveled change during successive time intervals of 0.1 s each?
- Horizontal motion of a projectile launched at an angle:

$$v_x = v_{x,i} = v_i \cos \theta = \text{constant}$$

$$\Delta x = (v_i \cos \theta) \Delta t$$



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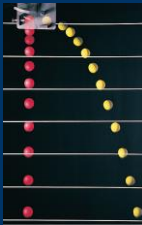
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Analysis of Projectile Motion

- Vertical motion is simple free fall.
  - Acceleration ( $a_y$ ) is a constant  $-9.81 \text{ m/s}^2$ .
  - Vertical velocity changes.
- How would the vertical distance traveled change during successive time intervals of 0.1 seconds each?
- Vertical motion of a projectile launched at an angle:



$$v_{y,f} = v_i \sin \theta + a_y \Delta t$$

$$v_{y,f}^2 = v_i^2 (\sin \theta)^2 + 2a_y \Delta y$$

$$\Delta y = (v_i \sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

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Projectile Motion

Click below to watch the Visual Concept.

Visual Concept

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Projectile Motion - Special Case

Initial velocity is horizontal only ( $v_{i,y} = 0$ ).

**HORIZONTAL MOTION OF A PROJECTILE**

$$v_x = v_{x,i} = \text{constant}$$

$$\Delta x = v_x \Delta t$$

**VERTICAL MOTION OF A PROJECTILE THAT FALLS FROM REST**

$$v_{y,f} = a_y \Delta t$$

$$v_{y,f}^2 = 2a_y \Delta y$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2$$

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### Projectile Motion Summary

- Projectile motion is free fall with an initial horizontal speed.
- Vertical and horizontal motion are independent of each other.
  - Horizontally the velocity is constant.
  - Vertically the acceleration is constant ( $-9.81 \text{ m/s}^2$ ).
- Components are used to solve for vertical and horizontal quantities.
- Time is the same for both vertical and horizontal motion.
- Velocity at the peak is purely horizontal ( $v_y = 0$ ).

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### Classroom Practice Problem (Horizontal Launch)

- People in movies often jump from buildings into pools. If a person jumps horizontally by running straight off a rooftop from a height of 30.0 m to a pool that is 5.0 m from the building, with what initial speed must the person jump?
- Answer: 2.0 m/s

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### Classroom Practice Problem (Projectile Launched at an Angle)

- A golfer practices driving balls off a cliff and into the water below. The edge of the cliff is 15 m above the water. If the golf ball is launched at 51 m/s at an angle of  $15^\circ$ , how far does the ball travel horizontally before hitting the water?
- Answer:  $1.7 \times 10^2 \text{ m}$  (170 m)

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### Now what do you think? -

- Suppose two coins fall off of a table simultaneously. One coin falls straight downward. The other coin slides off the table horizontally and lands several meters from the base of the table. ▾
  - Which coin will strike the floor first? ▾
  - Explain your reasoning. ▾
- Would your answer change if the second coin was moving so fast that it landed 50 m from the base of the table? Why or why not?

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### What do you think? -

- One person says a car is traveling at 10 km/h while another states it is traveling at 90 km/h. Both of them are correct. How can this occur? ▾
- Consider the frame of reference. ▾
  - Suppose you are traveling at a constant 80 km/h when a car passes you. This car is traveling at a constant 90 km/h. How fast is it going, relative to your frame of reference? How fast is it moving, relative to Earth as a frame of reference?

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### Relative Motion ▾

- Velocity differs in different frames of reference. ▾
- Observe as your instructor walks across the front of the room at a steady speed and drops a tennis ball during the walk. ▾
  - Describe the motion of the ball from the teacher's frame of reference. ▾
  - Describe the motion of the ball from a student's frame of reference. ▾
  - Which is the correct description of the motion?

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### Relative Motion

Click below to watch the Visual Concept.

Visual Concept

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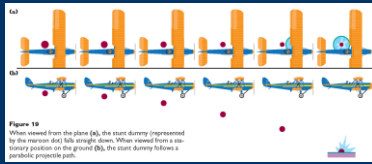
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### Frames of Reference

- A falling object is shown from two different frames of reference:
  - the pilot (top row)
  - an observer on the ground (bottom row)



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### Relative Velocity

- $V_{ac} = V_{ab} + V_{bc}$ 
  - $V_{ac}$  means the velocity of object "a" with respect to frame of reference "c"
  - Note:  $V_{ac} = -V_{ca}$
- When solving relative velocity problems, follow this technique for writing subscripts.

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## Sample Problem ▾

- A boat is traveling downstream. The speed of the boat with respect to Earth ( $v_{be}$ ) is 20 km/h. The speed of the river with respect to Earth ( $v_{re}$ ) is 5 km/h. What is the speed of the boat with respect to the river? ▾

- Solution: ▾

$$v_{br} = v_{be} + v_{er} = v_{be} + (-v_{re}) = 20 \text{ km/h} + (-5 \text{ km/h}) \quad \nabla$$

$$v_{br} = 15 \text{ km/h}$$

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## Classroom Practice Problem ▾

- A plane flies northeast at an airspeed of 563 km/h. (Airspeed is the speed of the aircraft relative to the air.) A 48.0 km/h wind is blowing to the southeast. What is the plane's velocity relative to the ground? ▾
- Answer: 565.0 km/h at  $40.1^\circ$  north of east ▾
- How would this pilot need to adjust the direction in order to maintain a heading of northeast? ▾

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## Now what do you think? ▾

- Suppose you are traveling at a constant 80 km/h when a car passes you. This car is traveling at a constant 90 km/h. ▾
  - How fast is it going, relative to your frame of reference? ▾
  - How fast is it moving, relative to Earth as a frame of reference? ▾
- Does velocity always depend on the frame of reference? ▾
- Does acceleration depend on the frame of reference? ▾

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